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Rotating superconductors: Ginzburg-Landau equations

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Abstract. Superconductors put into rotation develope a spontaneous internal magnetic field (the "London field"). In this paper Ginzburg Landau equations for order parameter, field, and current distributions for superconductors in rotation are derived. Two simple examples are discussed: the massive cylinder and the "Little and Parks geometry": a thin film of superconducting material deposited on a cylinder of normal material. A dependence of T_c on rotational frequency is predicted. The magnitude of the effect is estimated and should be observable.

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1 Introduction

Perfect electric conductivity and perfect diamagnetic susceptibility (Meissner effect) are the most striking macroscopic quantum phenomena in superconductors. Another consequence of superconductivity is the spontaneous appearance of a magnetic field inside the bulk of the superconductor – even without external field – if the superconductor is put into rotation. This phenomenon is not so well known and studied as the other two and its details are still subjects of active research. We discuss ideal sample shapes: cylinders or spheres. If rotated around their axis a spontaneous magnetic field inside the superconductor (usually called the "London field") is created:

$$\mathbf{B}_L = 2\frac{mc}{e}\boldsymbol{\omega} \tag{1}$$

where $\boldsymbol{\omega}$ is the angular velocity, $-e$ the electron charge, m its bare mass, and c is the velocity of light. Instead of expelling external magnetic fields from its interior as is the case in the Meissner effect, here the superconductor creates an internal field without an external field. Equation (1) does not contain material dependent parameters and applies to bulk superconductors (traditional as well as high T_c), the dimensions of which are large compared to the London penetration depth. A further requirement for its applicability is that the rotational velocity is low, such that the field $\mathbf{B}$ is smaller than the lower critical field and no vortices appear. Even slightly before the discovery of the Meissner-Ochsenfeld effect in 1933 Becker, *et al.* [1] predicted that a sphere of a perfect conductor put into rotation would generate a magnetic field of the correct size as given in equation (1). London arrived at the same conclusion based on the analysis of the London equations in the rotating frame of reference [2]. The basic cause for the effect is contained in the following qualitative arguments: Let ψ be the superconducting order parameter, which we can interpret as the macroscopic wave function for Cooper pairs,

$$\psi = |\psi| \exp\{i\phi\}; \quad \rho_s = |\psi|^2 \tag{2}$$

ρ_s being their density; $-2e$ is their charge, and $2m$ their bare mass (later on an effective mass will be defined as well). The momentum operator $\mathbf{p}_s$ for Cooper pairs and their velocity operator $\mathbf{v}_s$ are related by

$$2m\mathbf{v}_s = \mathbf{p}_s + \frac{2e}{c}\mathbf{A} \tag{3}$$

where $\mathbf{A}$ is the vector potential ($\mathbf{B} = \mathrm{rot}\,\mathbf{A}$). Without external magnetic fields and without rotation the phase ϕ of the macroscopic wave function ψ can be taken to be constant, $\mathbf{A}$ vanishes (we use the "London" gauge $\mathrm{div}\mathbf{A} = 0$ throughout) and the operator $\mathbf{p}_s = \hbar/\mathrm{i}\cdot\boldsymbol{\nabla}$ can be put equal to zero. Under rotation a magnetic field will appear, but for low rotational velocities the macroscopic wave function will remain stiff, that is will not change, in particular the gradient of ϕ will remain to be zero. Since the condensate described by the macroscopic wave function is charged its velocity under these conditions will be

$$\mathbf{v}_s = \frac{e}{mc}\mathbf{A}. \tag{4}$$

Rotating the normal part of the system (the ions and those electrons not being part of the condensate) with the velocity $\boldsymbol{\omega} \times \mathbf{r}$ amounts to creating a normal current $\mathbf{j}_n$. If the condensate would not rotate together with the normal fraction, large magnetic fields would result. To minimize the total free energy the contribution of the magnetic field

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must be limited; to achieve this the condensate will rotate with the same velocity as the normal fraction in the bulk, such that the total current density vanishes there. The condition

$$\mathbf{v}_{\rm s} = \boldsymbol{\omega} \times \mathbf{r} \ \text{ (in the bulk)} \tag{5}$$

together with equation (4) results in the vector potential

$$\mathbf{A} = \frac{mc}{e}\boldsymbol{\omega} \times \mathbf{r}, \tag{6}$$

which leads to the value of magnetic field given in equation (1). The necessary currents to create this field are flowing in a surface layer only, where the velocity of the normal part and the superconducting velocity $\mathbf{v}_{\rm s}$ are different (details about these surface currents will be given later). To finish these qualitative arguments we remark that the bare mass m appears in the expressions for vector potential $\mathbf{A}$ and magnetic field $\mathbf{B}$, since the velocity $\mathbf{v}_{\rm s}$ contains the bare mass and not some effective mass. Later on we will encounter an effective mass when discussing the details of the surface currents [3]. High precision measurements of the London field have been carried out aiming to determine relativistic corrections to the "Cooper pair mass" [4] in niobium. The experimental result, however, disagreed in order of magnitude and sign (!) with theoretical predictions. The cause for this discrepancy is still unclear [5]. Recently the energy of magnetic vortices in rotating superconductors has been discussed in relation to the astrophysical subject of rotating neutron stars [6], where the nuclear equations of state suggest the possibility of the condensation of protons to Cooper pairs and protonic superconductivity. In the following chapters Ginzburg-Landau equations for superconductors under rotation will be derived.

2 Free energy functional

The objective is to derive an appropriate expression for the free energy of a superconductor under rotation and obtain the Ginzburg–Landau equations through variation with respect to order parameter ψ and vector potential $\mathbf{A}$. First we have to clarify what is the physical constraint to be implemented to correctly describe the meaning of "under rotation". For the normal (= nonsuperconducting) state we can take the following view: We want to describe explicitly only the normal Fermi liquid of electrons, which is contained in a "reservoir" of ions rotating with a given angular velocity $\boldsymbol{\omega}$. The rotation of the ions will drag along the normal Fermi liquid through friction forces and impose to them the angular velocity $\boldsymbol{\omega}$ as well. The appropriate thermodynamic potential then can be obtained in the usual way from an effective Hamiltonian $H_{\rm eff}$, defined in the rotating frame of reference

$$\Omega = -k_{\rm B}T \cdot \ln(\mathrm{Tr}\exp(-\beta H_{\rm eff})). \tag{7}$$

Neglecting centrifugal forces and distortions of the lattice the effect of rotation is expressed in $H_{\rm eff}$ by an additional apparent vector potential [6]:

$$\mathbf{A}_{\rm rot} = -\frac{mc}{e}\boldsymbol{\omega} \times \mathbf{r}. \tag{8}$$

On the phenomenological level this amounts to the following. Let F_0 be the free energy in the fixed system of coordinates ("laboratory frame"); the subscript 0 refers to the normal state, *i.e.* for vanishing superconducting order parameter ψ. The natural variable for F_0 besides temperature T and volume V is the angular momentum $\mathbf{L}$ [7]. In the following the discussion will be restricted to constant volume V. The appropriate thermodynamic potential at given angular velocity $\boldsymbol{\omega}$ is obtained *via* a Legendre transformation

$$\mathrm{d}F_0 = -S\mathrm{d}T + \boldsymbol{\omega}\mathrm{d}\mathbf{L}, \quad G_0 = F_0 - \boldsymbol{\omega}\,\mathbf{L},$$
$$\mathrm{d}G_0 = -S\mathrm{d}T - \boldsymbol{L}\mathrm{d}\boldsymbol{\omega}. \tag{9}$$

For the construction of the free energy density in the superconducting state we use the notion of ψ being a macroscopic wavefunction for Cooper pairs, with charge density $\rho_{{\rm s},c}$ and bare mass density $\rho_{{\rm s},m}$:

$$\rho_{{\rm s},c} = -2e|\psi|^2, \;\; \rho_{{\rm s},m} = 2m|\psi|^2 \,. \tag{10}$$

The rest of the conduction electron system (we will call it the normal part) has mass density $\rho_{n,m}$ and charge density $\rho_{n,c}$; the ionic charge density is $\rho_{{\rm I},c}$, its current density $\mathbf{j}_{\rm I}$. We imply local charge neutrality:

$$\rho_{n,c} + \rho_{{\rm s},c} = \rho_{\rm I}. \tag{11}$$

Below we will use the kinetic energy and the angular momentum of the normal part:

$$E_{n,\rm kin} = \int \frac{1}{2}\rho_{n,m}\mathbf{v}_n^2\ \mathrm{d}V;$$
$$\boldsymbol{L}_n = \int \rho_{n,m}\boldsymbol{r} \times \boldsymbol{v}_n\ \mathrm{d}V. \tag{12}$$

$\mathbf{v}_n$ is the velocity of the normal fraction. In the superconducting state the constraint of given angular velocity $\boldsymbol{\omega}$ applies to the normal fraction of the system only: The ionic velocity $\mathbf{v}_{\rm I} = \boldsymbol{\omega} \times \mathbf{r}$ is controlled externally. Through friction forces the ions impose the same velocity to the *normal* fraction of the electronic system:

$$\mathbf{v}_n = \mathbf{v}_{\rm I}. \tag{13}$$

The velocity distribution of the superconducting fraction will then follow from the minimization of the potential to be formulated, containing kinetic energy of the superconducting part, magnetic field energy, and the electromagnetic coupling to the normal fraction and ions. This superconducting velocity distribution will not necessarily be identical to that of the normal fraction as illustrated in Figures 1 and 2:

Already Maxwell's equation will impose an equation of the type

$$\lambda^2 \cdot \boldsymbol{\nabla} \times \boldsymbol{h} = -(\frac{\hbar c}{2e}\boldsymbol{\nabla}\phi + \mathbf{A} - \frac{mc}{e}\boldsymbol{\omega} \times \mathbf{r}). \tag{14}$$

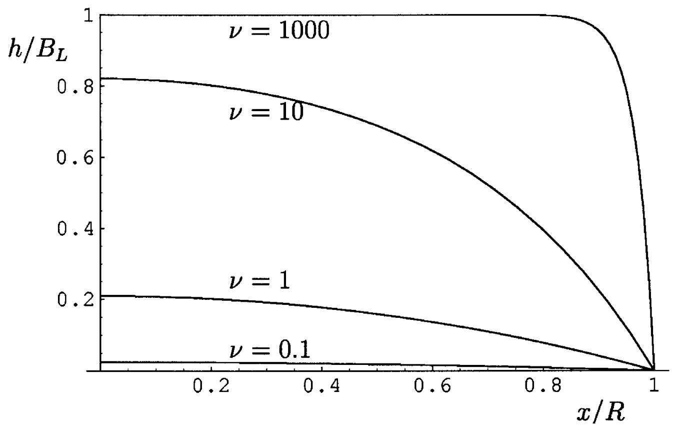

Fig. 1. The ratio of the internal magnetic field h to the hypothetical London field ($\mathbf{B}_L = 2mc/e\,\boldsymbol{\omega}$) as a function of the distance x from the center of the cylinder. R is the radius. $\nu = (R/\lambda)^2$, where λ is the penetration depth.

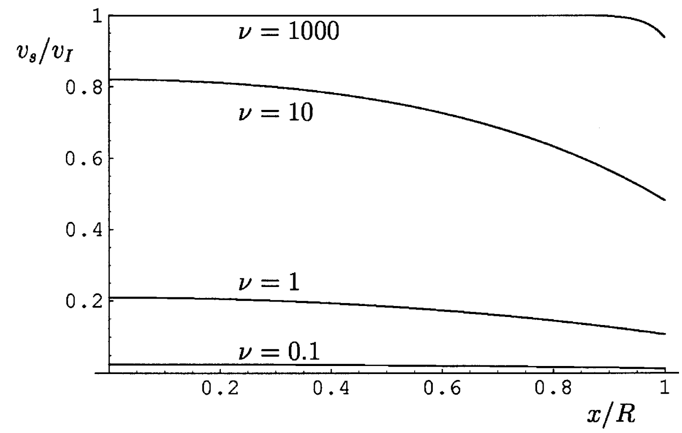

Fig. 2. The ratio of the superconducting velocity v_s to the ionic velocity v_I as a function of the distance x from the center of the cylinder.

Its solution for the geometry of long rotating cylinders, vanishing external magnetic field, and ideal "London field" (given by Eq. (1)) smaller than H_{c1} (implying $\boldsymbol{\nabla}\phi = 0$) are plotted: Figure 1 shows the ratio of the physical magnetic fields to the hypothetical "London field" as a function of distance from the center of the cylinder for different values of λ/R, where R is the radius. Only for small λ does the true field really reach the "London field" in the interior of the cylinder, whereas for λ/R of order 1 and smaller the resulting fields are much smaller throughout and as a consequence the velocity of the superconducting fraction (Eq. (4)) becomes much smaller than that of the normal fraction (as shown in Fig. 2).

For the construction of the thermodynamic potential we start from the free energy F written as the sum of the following contributions:

$$F = F_{0,0} + E_{\text{cond}} + E_{n,\text{kin}} + E_{s,\text{kin}} + E_W + E_{\mathbf{h}}. \quad (15)$$

The different terms are:
$F_{0,0}$ is the free energy in the normal state without rotation. E_{cond} is the condensation energy taken in its usual form:

$$E_{\text{cond}} = \int (\alpha|\psi|^2 + \frac{\beta}{2}|\psi|^4)\ \mathrm{d}V. \quad (16)$$

$E_{n,\text{kin}}$ is the additional kinetic energy of the normal part due to rotation (Eq. (12)). $E_{s,\text{kin}}$ is the kinetic energy of the superconducting fraction; using the macroscopic wave function ψ this has the form

$$E_{s,\text{kin}} = \langle\psi|\frac{1}{4m}(\frac{\hbar}{\mathrm{i}}\boldsymbol{\nabla} + \frac{2e}{c}\mathbf{A})^2|\psi\rangle = \int \frac{1}{4m}|(\frac{\hbar}{\mathrm{i}}\boldsymbol{\nabla} + \frac{2e}{c}\mathbf{A})\psi|^2\ \mathrm{d}V. \quad (17)$$

In the kinetic energy operator the bare mass m appears.

E_W is an interaction energy of the superconducting fraction with the rest of the system: For small velocities this should be proportional to the square of the relative velocities between superconducting and ionic part:

$$E_W = \gamma m\langle\psi|(\mathbf{v}_\text{s} - \mathbf{v}_\text{I})^2|\psi\rangle = \int \frac{\gamma}{4m}|(\frac{\hbar}{\mathrm{i}}\boldsymbol{\nabla} + \frac{2e}{c}\mathbf{A} - 2m(\boldsymbol{\omega}\times\mathbf{r}))\psi|^2\ \mathrm{d}V. \quad (18)$$

γ is a material dependent parameter, it will be responsible for the introduction of a material dependent effective mass m^* into the problem [3].

$E_{\mathbf{h}}$ is the energy of the magnetic field:

$$E_{\mathbf{h}} = \int \frac{\mathbf{h}^2}{8\pi}\ \mathrm{d}V. \quad (19)$$

Now the transformation from free energy F to thermodynamic potential G: There are two constraints to be implemented: The first concerns the kinetic energy of the normal fraction, the second the magnetic field.

The angular velocity of the normal fraction must be identical to the ionic velocity (Eq. (13)), this constraint leads to a term $-\boldsymbol{\omega}\cdot\mathbf{L}_n$. The formal procedure is contained in the following arguments: For normal systems $G_0 = F_0 - \boldsymbol{\omega}\mathbf{L}$ is the thermodynamic potential: The free energy F_0, which is increased by finite kinetic energy due to rotation, has to be at a minimum under the additional constraint, that the angular velocity is equal to $\boldsymbol{\omega}$, imposed to the system due to the coupling to the "reservoir". Formally we might view the term $-\boldsymbol{\omega}\mathbf{L}$ as resulting from the constraint being imposed through a Lagrange parameter, the physical significance of the Lagrange parameter being the total angular momentum. The explicit coupling to the "reservoir" is not specified in the thermodynamic description, the only condition being that this coupling fixes the angular velocity. For the specific system of a normal Fermi liquid the coupling would be due to friction forces between the "reservoir" (consisting of ions) and the electrons, resulting in identical velocities of electrons and "reservoir" in equilibrium.

In the presence of superconductivity expressed through the additional thermodynamic variable ψ the description becomes more detailed. The interaction of that part of the system which is described by ψ to the rest of the system is contained explicitly in our thermodynamic potential: This coupling is through electromagnetic forces, the magnetic field energy and the coupling of ψ to the magnetic field (through the charge) are described explicitly in the formulation of the free energy. Friction forces between ψ and the rest of the system do not exist and the angular velocity of the superconducting fraction is not required to be identical to that of the rest of the system (as demonstrated explicitly in the figures). The constraint to be imposed refers to the normal part of the system only: Finite velocity $\mathbf{v}_{\rm I}$ of the ions (which constitute our "reservoir") fixes the velocity $\mathbf{v}_{\rm n}$ of the normal part of the electronic system only. Formally this results in the term $-\boldsymbol{\omega}\mathbf{L}_n$ in the thermodynamic potential. Again we might view this term to result from a constraint imposed through a Lagrange parameter $(-\int \mathbf{v}_n \cdot \mathbf{Y}\ {\rm d}V)$. The physical significance of the Lagrange parameter $\mathbf{Y}$ is obtained by requiring that the derivative of $(F - \int \mathbf{v}_n \cdot \mathbf{Y}\ {\rm d}V)$ with respect to $\mathbf{v}_n$ vanishes, which leads to

$$\int \mathbf{v}_n \cdot \mathbf{Y}\ {\rm d}V = \boldsymbol{\omega} \cdot \mathbf{L}_n. \tag{20}$$

What enters is the angular momentum of the normal fraction only and not the total angular momentum. The angular velocity of the superconducting fraction, its spatial variation, as well as the spatial variations of ψ and magnetic field $\mathbf{h}$, will result from an optimization of kinetic energy, magnetic field energy, and condensation energy contained explicitly in the free energy.

Furthermore we have to make the transition from the variable magnetic induction $\mathbf{B}$ (which is the average of $\mathbf{h}$) to the natural variable $\mathbf{H}$, where $\boldsymbol{\nabla} \times \mathbf{H} = \frac{4\pi}{c}\mathbf{J}$. $\mathbf{J}$ is the current density which is controlled externally, for our problem this is the ionic current density $\mathbf{j}_{\rm I}$. Further contributions to $\mathbf{J}$ can be those external currents, which create an externally applied field (if present), in this paper we discuss examples only where no external field is applied. What is slightly unusual here is that the ionic currents which are controllable "externally" (contributing to the field $\mathbf{H}$) are actually flowing inside the sample.

We obtain the potential G:

$$G = F - \boldsymbol{\omega} \cdot \mathbf{L}_n - \int \frac{\mathbf{h} \cdot \mathbf{H}}{4\pi}\ {\rm d}V. \tag{21}$$

This may also be written as:

$$\begin{aligned} G(T,V,\boldsymbol{\omega}) \ =\ & G_0(T,V,\boldsymbol{\omega}) + \int m|\psi|^2(\boldsymbol{\omega} \times \mathbf{r})^2\ {\rm d}V \\ & + \int (\alpha|\psi|^2 + \frac{\beta}{2}|\psi|^4)\ {\rm d}V + \int \frac{(\mathbf{h}^2 - 2\mathbf{h} \cdot \mathbf{H})}{8\pi}\ {\rm d}V \\ & + \int \frac{1}{4m}|(\frac{\hbar}{\rm i}\boldsymbol{\nabla} + \frac{2e}{c}\mathbf{A})\psi|^2\ {\rm d}V \\ & + \int \frac{\gamma}{4m}|(\frac{\hbar}{\rm i}\boldsymbol{\nabla} + \frac{2e}{c}\mathbf{A} - 2m(\boldsymbol{\omega} \times \mathbf{r}))\psi|^2\ {\rm d}V. \end{aligned} \tag{22}$$

G_0 is the thermodynamic potential in the normal state (see Eq. (9)).

3 Ginzburg-Landau equations

To obtain the Ginzburg-Landau equations, variations with respect to $\mathbf{A}$ and ψ are performed. The equation obtained from the variation with respect to $\mathbf{A}$ takes the form

$$\begin{aligned} \mathbf{j} = \frac{\boldsymbol{\nabla} \times \mathbf{h}}{(4\pi)/c} = & -\frac{\hbar e}{{\rm i}2m^*}(\psi^* \boldsymbol{\nabla}\psi - \psi\boldsymbol{\nabla}\psi^*) \\ & + \frac{e}{m^*}|\psi|^2(-\frac{2e}{c}\mathbf{A} + 2m(\boldsymbol{\omega} \times \mathbf{r})), \end{aligned} \tag{23}$$

where the effective mass m^* has been defined

$$m^* = \frac{m}{1+\gamma}\ \cdot \tag{24}$$

The equation for the current has the required form of equation (14). The material dependent parameter γ affects the characteristic magnetic length λ (London penetration depth) but not the parameter bare m, which appears in the value of the London field (the bare m appears inside the bracket of the last term on the right hand side of equation (23). Relativistic corrections are not considered in this paper.

The equation obtained through variation with respect to ψ (the "first G-L equation") is

$$\begin{aligned} & (\alpha + m(\boldsymbol{\omega} \times \mathbf{r})^2)\psi + \beta|\psi|^2\psi + \frac{1}{4m}(\frac{\hbar}{\rm i}\boldsymbol{\nabla} + \frac{2e}{c}\mathbf{A})^2\psi \\ & \quad + \frac{\gamma}{4m}(\frac{\hbar}{\rm i}\boldsymbol{\nabla} + \frac{2e}{c}\mathbf{A} - 2m(\boldsymbol{\omega} \times \mathbf{r}))^2\psi = 0. \end{aligned} \tag{25}$$

An equivalent form for this equation is

$$\begin{aligned} & \alpha\psi + \beta|\psi|^2\psi + (\boldsymbol{\omega} \times \mathbf{r}) \cdot (\frac{\hbar}{\rm i}\boldsymbol{\nabla} + \frac{2e}{c}\mathbf{A})\psi \\ & \quad + \frac{1}{4m^*}(\frac{\hbar}{\rm i}\boldsymbol{\nabla} + \frac{2e}{c}\mathbf{A} - 2m(\boldsymbol{\omega} \times \mathbf{r}))^2\psi = 0. \end{aligned} \tag{26}$$

The existence of the third term on the left hand side of equation (26) is a direct result of equation (20): It was shown there that the transformation from free energy F to thermodynamic potential G for superconductors contains the term $\boldsymbol{\omega} \cdot \mathbf{L}_n$ and not the full angular momentum $\mathbf{L}$, as would be the case for a normal system. As a consequence the formal equivalence of rotation and magnetic field, as partly still contained in the forth term of the left hand side of equation (26), is partly lifted for superconductors. If instead of $\mathbf{L}_n$ the full $\mathbf{L}$ is used the third term on the left hand side would be absent, the Ginzburg-Landau equation would recover the form used in an early publication of Verkin and Kulik [8].

4 Consequences

Let us first consider a bulk cylinder of radius R large compared to the London penetration depth λ, rotating at an angular velocity $\boldsymbol{\omega}$ small enough such that the resulting field $\mathbf{B}$ is smaller than the lower critical field $\mathbf{H}_{c1}$. For this situation the macroscopic wave function ψ remains essentially "stiff" (= unaltered from its value at vanishing $\boldsymbol{\omega}$), corrections being of negligible importance for the solution to the current equation (23). The term containing the gradient of the phase ϕ of the order parameter ψ vanishes and the current equation takes the simple form

$$\boldsymbol{\nabla} \times (\boldsymbol{\nabla} \times \mathbf{h}) = -\frac{1}{\lambda^2}(\mathbf{h} - \frac{2mc}{e}\boldsymbol{\omega}), \tag{27}$$

where the length λ is given by

$$\frac{1}{\lambda^2} = \frac{4\pi e^2}{c^2}\frac{|\psi|^2}{m^*}\,. \tag{28}$$

For vanishing externally applied field the transition from vanishing field at the surface towards the internal field $\mathbf{B} = \boldsymbol{\omega}\ 2mc/e$ occurs within the characteristic length λ: Using cylindrical coordinates (x being the distance from the axis of the cylinder, and z the coordinate along the axis), $\mathbf{h}$ has only a z-component depending on t, where $t = x/R$. The solution is given in terms of the modified Bessel function I_0:

$$h_z(t) = \frac{2mc}{e}\omega\frac{I_0(\sqrt{\nu}) - I_0(\sqrt{\nu}t)}{I_0(\sqrt{\nu})}, \tag{29}$$

where $\nu = (R/\lambda)^2$. The solutions for various values of R/λ are plotted in Figure 1. For R/λ large the field increases approximately in exponential fashion from zero at the surface towards the London field $\boldsymbol{\omega}\ 2mc/e$ in the bulk:

$$\mathbf{h} \approx (1 - \exp(\frac{x-R}{\lambda}))\frac{2mc}{e}\boldsymbol{\omega}. \tag{30}$$

The physical interpretation is straightforward: whereas inside the bulk the velocities of the normal and superconducting fractions are equal and hence the total current density vanishes, the velocities differ in the surface layer. There the superconducting fraction lags behind the normal one and the total current density is finite. Starting from zero at the surface the magnetic field $\mathbf{h}$ increases towards its bulk value over the characteristic length λ, just in the same way as the total current density (proportional to $\mathbf{v}_n - \mathbf{v}_\mathrm{s}$) drops from its maximal value at the surface towards zero in the bulk.

A final remark concerning the length λ: It contains the ratio $|\psi|^2/m^*$, with the material dependent effective mass m^*, different from the bare mass m (there had been some confusion about this distinction in some early papers on the London field, which was cleared up in Ref. [3]), whereas the bare mass appears in the London field $\mathbf{B}$. As a consequence, a measurement of λ does not fix the scale of ψ, since this measures only a ratio, which contains another material dependent parameter.

As a second example we discuss the geometry of the "Little and Parks Experiment" [9,10]. We consider a thin-walled superconducting cylinder: A thin film of superconducting material of thickness d is deposited on a cylinder of normal material of radius R, with $d \ll R$. Let us further restrict the discussion to the situation where d is also small compared to λ as well as coherence length ξ (the situation discussed by de Gennes [10]). The current equation takes the form

$$\mathbf{j} = -\frac{e}{m^*}|\psi|^2(\hbar\boldsymbol{\nabla}\phi + \frac{2e}{c}\mathbf{A} - 2m\boldsymbol{\omega} \times \mathbf{R}). \tag{31}$$

The superconducting velocity $\mathbf{v}_\mathrm{s}$ now is

$$\mathbf{v}_\mathrm{s} = \frac{1}{2m}(\hbar\boldsymbol{\nabla}\phi + \frac{2e}{c}\mathbf{A}), \tag{32}$$

and equation (31) takes the form

$$\mathbf{j} = -\frac{2me}{m^*}|\psi|^2(\mathbf{v}_\mathrm{s} - \boldsymbol{\omega} \times \mathbf{R}). \tag{33}$$

The first Ginzburg-Landau equation becomes

$$(\alpha + 2m\mathbf{v}_\mathrm{s} \cdot (\boldsymbol{\omega} \times \mathbf{R}))\psi + \beta|\psi|^2\psi + \frac{(m)^2}{m^*}(\mathbf{v}_\mathrm{s} - \boldsymbol{\omega} \times \mathbf{R})^2\psi = 0. \tag{34}$$

In contrast to the Little and Parks experiment, where an external magnetic field is applied, here we have finite rotational velocity. In the former case special values of magnetic fields – corresponding to integer multiples of magnetic flux through the cylinder – existed for which the quantized integral of $\boldsymbol{\nabla}\phi$ around the perimeter lead to vanishing velocity $\mathbf{v}_\mathrm{s}$. For these special magnetic field values the transition temperature T_c was unchanged, while for intermediate fields $\mathbf{v}_\mathrm{s}$ was finite and T_c suppressed, T_c became a periodic function of magnetic flux. For the effect to be observable the radius had to be small, since $\mathbf{v}_\mathrm{s}$ is proportional to $1/R$.

Now for finite ω and vanishing external field again there will be special values of rotational frequency ω_n:

$$\omega_n = \frac{\hbar n}{2mR^2}\,, \tag{35}$$

where the integer n is given by $\oint \boldsymbol{\nabla}\phi \cdot \mathrm{d}\mathbf{l} = 2\pi n$. For these special values ω_n, $(\mathbf{v}_\mathrm{s} - \boldsymbol{\omega}_n \times \mathbf{R})$ vanishes, there is no current and no magnetic field. For intermediate values of ω finite currents exist resulting in finite magnetic flux (the scenario studied in the high precision experiment of reference [4]).

We obtain a dependence of T_c on rotational frequency: For $\omega = \omega_n$ the last term on the left hand side of equation (34) vanishes. Equation (34) predicts a lowering of T_c due to the additional term $2m\mathbf{v}_s \cdot (\boldsymbol{\omega}_n \times \mathbf{R}) = 2m\omega_n^2 R^2$. An estimate for the order of magnitude of this effect: for $\omega/2\pi$ of order 10^2 s^{-1} and R of order 0.1 m a shift in transition temperature of order 0.1 K is obtained for tin (Sn, that was the material of the original Little and Parks experiment [9] and direct comparison can be made): the effect should be observable.

A further remark concerning intermediate values of ω: Although in principle T_c according to equation (34) should be a periodic function of frequency due to the last terms on the left hand side, the amplitudes of these oscillations are too small for observation.

Higher order corrections in d/λ , d/ξ , λ/R , ξ/R will not be discussed in this paper. These, however, should be considered for a reanalysis of the high precision experiment [4]: including these corrections might reconcile experiment and theory.

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References

1. R. Becker, G. Heller, F. Sauter, Z. Phys. **85**, 772 (1933).
2. F. London, *Superfluids*, Vol. 1 (Dover, New York, 1961).
3. J.A. Geurst, H. van Beelen, Physica C **208**, 43 (1993).
4. J. Tate, B. Cabrera, S.B. Felch, J.T. Andersen, Phys. Rev. Lett. **62**, 845 (1989).
5. references in: M. Liu, Phys. Rev. Lett. **81**, 3223 (1998); Y. Jiang, M. Liu, Phys. Rev. B **63**, 184506 (2001).
6. B. Carter, R. Prix, D. Langlois, Phys. Rev. B **62**, 9740 (2000); B. Carter, D. Langlois, R. Prix, Phys. Rev. B **62**, 9748 (2000).
7. L.D. Landau, E.M. Lifshitz, *Course of Theoretical Physics*, Vol. 5 (*Statistical Physics*), Chapts. 26 and 34; and Vol. 1 (*Mechanics*) Chap. 39.
8. B.I. Verkin, I.O. Kulik, Sov. Phys. JETP **34**, 1103 (1972).
9. W.A. Little, R.D. Parks, Phys. Rev. Lett. **9**, 9 (1962); Phys. Rev. A **133**, 97 (1964).
10. P.G. de Gennes, *Superconductivity of Metals and Alloys* (Benjamin Press, 1966).